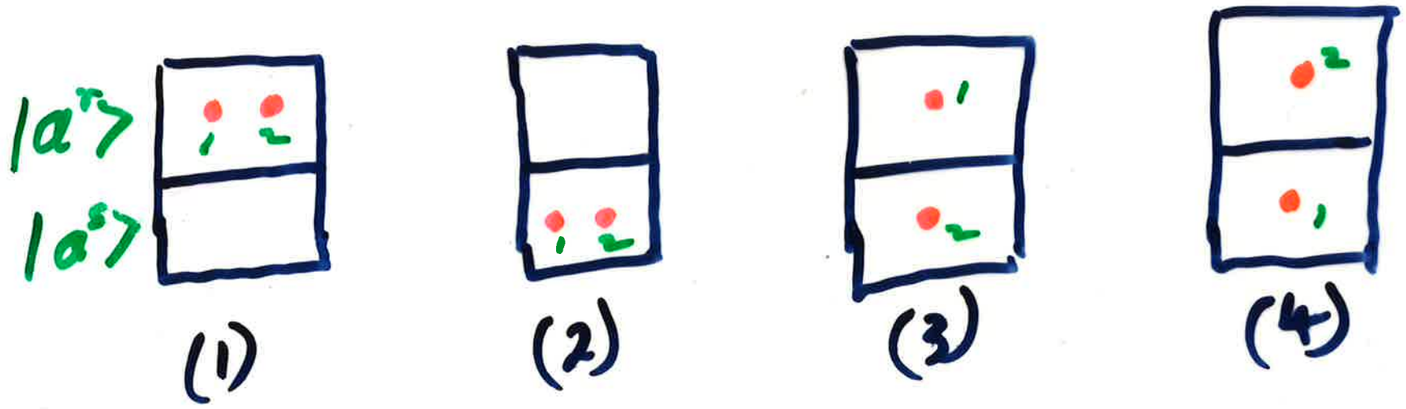
PRINCIPLE OF THE IDENTITY OF INDISCERNIBLES

$$\forall F(F(\underline{a}) \leftrightarrow F(\underline{b})) \rightarrow \underline{a} = \underline{b}$$

Weak Version: F includes properties & spatial location

Strong Version: F does not include properties of spatial location

Quantum Statistics



State vectors:

(1) $|a^r\rangle \otimes |a^r\rangle$

(2) $|a^s\rangle \otimes |a^s\rangle$

(3) $|a^r\rangle \otimes |a^s\rangle$

(4) $|a^s\rangle \otimes |a^r\rangle$

New Basis for subspace

(5) $|a^1\rangle \otimes |a^2\rangle$

(6) $|a^3\rangle \otimes |a^3\rangle$

(7) $\frac{1}{\sqrt{2}}(|a^2\rangle \otimes |a^3\rangle + |a^3\rangle \otimes |a^2\rangle)$

(8) $\frac{1}{\sqrt{2}}(|a^2\rangle \otimes |a^3\rangle - |a^3\rangle \otimes |a^2\rangle)$

INDISTINGUISABILITY POSTULATE

$$\langle P\phi|Q|P\phi\rangle = \langle\phi|Q|\phi\rangle$$

$$\forall\theta, \forall\phi$$

$|P\phi\rangle \underset{\text{def}}{=} P|\phi\rangle$, P is Unitary permutation operator on particle labels

As restriction on Q this implies $QP = PQ$, so Q must be a symmetric function of the particle labels.

PROPERTIES OF INDIVIDUAL SYSTEMS

$$Q_1 = Q \otimes I$$

$$Q_2 = I \otimes Q$$

Q_1 and Q_2 are not observables

But probability of observing z^d on one or other particle is

$$\text{Prob}(Q_1 = z^d) + \text{Prob}(Q_2 = z^d) - \text{Prob}(Q_1 = z^d \text{ & } Q_2 = z^d)$$

Monadic and relational properties

$$\text{Prob}(Q_1 = z^\alpha) = \text{Prob}(Q_2 = z^\alpha)$$

$$\text{Prob}(Q_1 = z^\alpha / Q_2' = z'^\beta) = \text{Prob}(Q_2 = z^\alpha / Q_1' = z'^\beta)$$

For 3-particle system we must consider also

$$\text{Prob}(Q_1 = z^\alpha / Q_2' = z'^\beta * Q_3'' = z''^\gamma)$$

$$= \text{Prob}(Q_3 = z^\alpha / Q_2' = z'^\beta * Q_1'' = z''^\gamma)$$

where
$$\left.\begin{aligned} Q_1 &= Q \otimes I \otimes I \\ Q_2 &= I \otimes Q \otimes I \\ Q_3 &= I \otimes I \otimes Q \end{aligned}\right\}$$